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LINE GRAPH ASSOCIATED WITH C-PRIME GRAPH OF A NEARRING

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ABSTRACT. In this paper we introduce a line graph of c-prime graph of a nearring N with respect to an ideal I denoted by $L(C_I(N))$. We find properties of this graph. We find diameter, girth and obtain conditions for $L(C_I(N))$ is a complete graph. We find interrelation between $L(G_I(N))$, $L(C_I(N))$. We prove that nearring homomorphism between two nearrings is a graph homomorphism between corresponding c-prime line graphs.

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1. INTRODUCTION

Algebra and graph theory are two independent fields of mathematics. Researchers tried to relate these fields and develop further study in these fields. By the introduction of zero divisor graphs of a ring by the authors in [2, 3] a new relation found between ring theory and graph theory. Redmond, et al. [17] further studied these graphs and obtained the size of a ring using the zero-divisor graph of the ring. Another type of the graph, namely the total graph of a ring introduced by the authors in [1]. These concepts are generalized by the authors in [5] by introducing the graph of a nearring with respect to an ideal. Chelvam et al. [7] further studied these graphs and introduced zero divisor graph using the ideal of nearring. Authors in [12, 14] studied interval valued L-fuzzy ideals and cosets.
In this paper, we introduce a line graph of the c-prime graph of nearring denoted by $L(C_I(N))$ and find properties of this graph. We find a relation between $L(C_I(N))$ and $L(G_I(N))$ and obtain conditions for $L(C_I(N))$ to be a complete graph. We find conditions for girth of $L(C_I(N))$ to be ∞, find the dominating set of the graph and obtain properties of this graph under homomorphism.

2. DEFINITIONS AND PRELIMINARIES

In this paper, N, N_1, N_2 represent a right nearring. We refer to Pilz [15], Ferrero and Ferrero [9] for nearrings, Harrary [10], Clark and Holton [8], Bhavnari and Kuncham [4] for topics on graph theory, Kedukodi et al. [13] and Bhavanari et al. [5] for graph of nearing with respect to ideal, Sarmah et al. [16] for line graph, Booth et al. [6] for prime ideals in nearrings, Hell [11] for graph homomorphism. Following definitions are from the book by Harary [10]
A *graph* $G = (V, E)$ consists of a set of objects $V = \{v_1, v_2, ...v_n\}$ called *vertices* (or points) and another set $E = \{e_1, e_2, ..e_n\}$ whose elements are called *edges* such that each edge e is identified with an unordered pair (v_i, v_j) of vertices. A graph

$G = (V, E)$ is said to be complete if $(u, v) \in E$ for all $u, v \in V, u \neq v$. A complete graph with n vertices is denoted by K_n. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are two graphs. A *bipartite graph* G is a graph whose point set V is partitioned into two subsets V_1 and V_2 such that every edge of G joins V_1 with V_2. If G contains every edge joining V_1 and V_2 then G is called a *complete bipartite graph*. If $|V_1| = m, |V_2| = n$ then G can be written as $K_{m,n}$. The graph $K_{1,n}$ is called a *star graph* with $V_1 = \{v_1\}$, here v_1 represents the *root vertex* of the star graph. Let x and y be distinct vertices of graph G and $d(x, y)$ be the length of shortest path between x and y ($d(x, y) = \infty$ if there is no such path). The *diameter of* G is denoted by $diam(G)$, given by $diam(G)= sup\ \{d(x, y) \mid\ x$ and y are distinct vertices of $G\}$. A *vertex cover* of graph G is a subset K of V such that if (u, v) is an edge of G, then $u \in K$ or $v \in K$ or both $u \in K$ and $v \in K$. The girth of G, denoted by *gr(G)*, is the length of a shortest cycle in G and $gr(G) = \infty$ if G contains no cycle.

3. Line Graph Associated with c-prime Graph of a Nearring

Definition 3.1. The *line graph* of $C_I(N)$ is a graph with edge set of $C_I(N)$ as vertex set and any two distinct vertices are adjacent in the line graph if and only if their corresponding edges share a common vertex in the graph $C_I(N)$. We denote this graph by $L(C_I(N))$.

Example 3.2. (1) Consider $\mathbb{Z}_3$, the ring of integers modulo 3. Ring $\mathbb{Z}_3$ has ideals $I_1 = \{0\}$, $I_2 = \mathbb{Z}_3$. The graphs $C_{I_1}(\mathbb{Z}_3)$ and $C_{I_2}(\mathbb{Z}_3)$ are shown in Figure 1 and corresponding line graphs are shown in Figure 2.
(2) Consider $\mathbb{Z}_6$ the ring of integers modulo 6. Consider the ideals of the ring $I_1 = \{0\}$, $I_2 = \{0, 3\}$. The graph $C_{I_1}(\mathbb{Z}_6)$ and $L(C_{I_1}(\mathbb{Z}_6))$ is shown in Figure 3. Also the graph $C_{I_2}(\mathbb{Z}_6)$ and $L(C_{I_2}(\mathbb{Z}_6))$ is shown in Figure 4.

$C_{I_1}(\mathbb{Z}_3)$ $C_{I_2}(\mathbb{Z}_3)$

Figure 1. Diagram of $C_I(\mathbb{Z}_3)$

Remark 3.3. 1. $L(C_I(N))$ is a *Connected Graph* without *self-loops* and *multiple edges*.
2. 0 is adjacent to every other vertex of $C_I(N)$. Hence $L(C_I(N))$ can never be a *Null Graph*.

Proposition 3.4. *Let N be a nearring. Then $L(C_I(N))$ is connected and $diam(L(C_I(N))) \leq 3$.*

Proof. As 0 is connected to all other vertices of $C_I(N)$ hence $L(C_I(N))$ is always connected. Let $a, b, c, d \in N$ such that $X = [a, b]$ and $Y = [c, d]$ are any two distinct vertices in $L(C_I(N))$. Suppose X and Y share a common vertex then X and Y are adjacent in $L(C_I(N))$. (That is if $a = c$ or $b = d$ then X and Y are adjacent.) Then $diam(L(C_I(N))) = 1$. Similar argument holds if $a = d$ or $b = c$. Suppose that X and Y do not have vertex in common. Then can construct a path $[a, b]-[a, 0]-[0, d]-[c, d]$

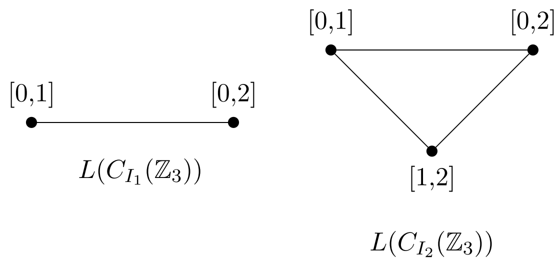


FIGURE 2. Diagram of $L(C_I(\mathbb{Z}_3))$

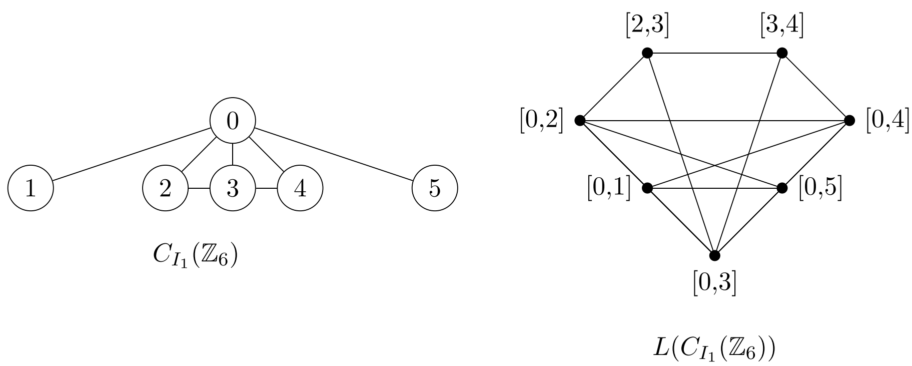


FIGURE 3. Diagram of $C_{I_1}(\mathbb{Z}_6)$ and $L(C_{I_1}(\mathbb{Z}_6))$

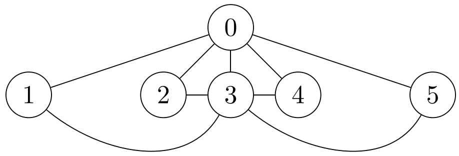
or $[a,b]-[0,b]-[0,d]-[c,d]$ of length 3. Then $diam(L(C_I(N)))=3$. Hence in any case $diam(L(C_I(N)))\leq 3$. □

Proposition 3.5. *Let N be a nearring, I be a 3-prime ideal of N. Then $L(G_I(N))\subseteq L(C_I(N))$.*

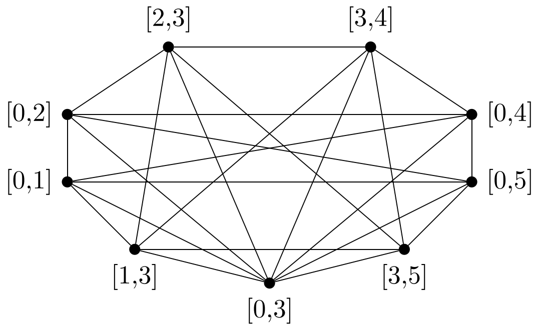
Proof. Let $a,b,c,d\in N$ such that $([a,b],[c,d])$ be an edge in $L(G_I(N))$. Then corresponding edges share a common vertex $G_I(N)$. Suppose that a is a common vertex say,

$$a=c \tag{1}$$

As $[a,b]$ and $[c,d]$ are vertices in $L(G_I(N))$. Then (a,b) and (c,d) are edges in $G_I(N)$. Then by the definition of $G_I(N)$ get, $aNb\subseteq I$ or $bNa\subseteq I$ and $cNd\subseteq I$ or $dNc\subseteq I$. Without loss of generality, may assume that $aNb\subseteq I$ and $cNd\subseteq I$. From Equation (1), get $aNb\subseteq I$ and $aNd\subseteq I$. As I is a *3-prime ideal* $a\in I$ or $b\in I$ and $c\in I$ or



$C_{I_2}(\mathbb{Z}_6)$



$L(C_{I_2}(\mathbb{Z}_6))$

FIGURE 4. Diagram of $C_{I_2}(\mathbb{Z}_6)$ and $L(C_{I_2}(\mathbb{Z}_6))$

$d \in I$. Suppose $a \in I$ and $c \in I$. Then $aN \subseteq I$(by the property of the ideal) implies $ab \in I$ and $cd \in I$ which implies $(a, b) \in E(C_I(N))$ and $(c, d) \in E(C_I(N))$. From Equation (1) get $([a, b], [c, d]) \in E(L(C_I(N)))$. Therefore $L(G_I(N)) \subseteq L(C_I(N))$. Suppose $a \in I$ and $d \in I$. Then, $a \in I$ which implies $ab \in I$ (by the property of an ideal). Similarly get $dc \in I$. Hence $(a, b) \in E(C_I(N))$ and $(d, c) \in E(C_I(N))$. By Equation 1 get $([a, b], [c, d]) \in E(L(C_I(N)))$. Hence $E(L(G_I(N))) \subseteq E(L(C_I(N)))$. By definition of $L(G_I(N))$ and $L(C_I(N))$ have $V(L(G_I(N))) \subseteq V(L(C_I(N)))$. Thus $L(G_I(N)) \subseteq L(C_I(N))$.

□

Now provide an example to show that the $\subseteq$ in above proposition can be strict even when I is an 3-prime ideal of N.

Example 3.6. Let N be a nearring $N = \{0, a, b, c\}$ under the binary operations $+$ and $\cdot$ defined by Table 1.

TABLE 1. Nearring for Example 3.6

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

·	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	a	b	c
c	a	0	c	b

Note that $I = \{0\}$ is a *3-prime ideal* of N. The graphs $L(G_I(N))$ and $L(C_I(N))$ is shown in Figure 6.

$G_I(N)$ $C_I(N)$

FIGURE 5. Diagram of $G_I(N)$ and $C_I(N)$

$L(G_I(N))$ $L(C_I(N))$

FIGURE 6. Diagram of $L(G_I(N))$ and $L(C_I(N))$

Observe that $L(G_I(N)) \subset L(C_I(N))$.

Proposition 3.7. *Let N be a nearring. If I is a c-prime ideal then $L(G_I(N)) = L(C_I(N)$.*

Proof. Let I be a *c-prime ideal* of N. As every *c-prime ideal* of N is a *3-prime ideal* get I is a *3-prime ideal* of N. By Proposition 3.5, get that $L(G_I(N)) \subseteq L(C_I(N)$. Let $a, b, c, d \in N$ such that $([a, b], [c, d]) \in E(L(C_I(N)))$. Then corresponding edges share a common vertex $C_I(N)$. Assume that a is a common vertex,

$$a = c \tag{2}$$

As $[a, b]$ and $[c, d]$ are vertices in $L(C_I(N))$. Then (a, b) and (c, d) are edges in $C_I(N)$. Then by definition of $C_I(N)$ get $ab \in I$ and $cd \in I$. As I is *c-prime* get that $a \in I$ or $b \in I$ and $c \in I$ or $d \in I$. Suppose $a \in I, c \in I$. Then $aN \subseteq I$ (by the property of an ideal) implies $aNb \subseteq Ib \subseteq I$ implies $(a, b) \in E(G_I(N))$. Similarly $(c, d) \in E(G_I(N))$. From Equation (2) get $([a, b], [c, d]) \in E(L(G_I(N)))$. Suppose $b \in I$ and $d \in I$. Then $bN \subseteq I$, get that $bNa \subseteq Ia \subseteq I$ implies $(b, a) \in E(G_I(N))$ and $dNc \subseteq Ic \subseteq I$ implies $(d, c) \in E(G_I(N))$. By Equation(2), get $([a, b], [c, d]) \in E(L(G_I(N)))$. Hence $E(L(C_I(N))) \subseteq E(L(G_I(N))$. Therefore $E(L(G_I(N))) = E(L(C_I(N))$. By definition of $L(G_I(N))$ and $L(C_I(N))$ have $V(L(C_I(N))) = V(L(G_I(N)))$. Thus $L(G_I(N)) = L(C_I(N)$. □

Remark 3.8. Now provide an example to show that the $L(G_I(N)) \neq L(C_I(N)$ when I is not a c-prime ideal of N. In Example 3.6, observe that $I = \{0\}$ is a 3-prime ideal of N and I is not a c-prime ideal of N (observe that $c \cdot a = 0 \in I$ but $c \notin I$ and $a \notin I$.) Note that $L(G_I(N)) \neq L(C_I(N)$.

Proposition 3.9. *Let N be an integral nearring with $|N| = n$, and $I = \{0\}$ be a c-prime ideal of N. Then $L(C_I(N))$ is a complete graph.*

Proof. Let N be an *integral nearring.* Then N has no non-zero zero divisors. In the graph $C_I(N)$ get 0 is adjacent to every other vertex. Suppose $a \neq 0$ and $b \neq 0$ such that a and b are adjacent in $C_I(N)$. Then $ab \in I$. As I is c-prime ideal of N get $a \in I = \{0\}$ or $b \in I = \{0\}$. Then $a = 0$ or $b = 0$ a contradiction. Hence $C_I(N)$ is a star Graph of degree n with 0 as center vertex. Let $N = \{0, x_1, x_2, ..., x_{n-1}\}$ then $V(L(C_I(N))) = \{[0, x_i] : 1 \leq i \leq n-1\}$. In the graph $L(C_I(N))$, $[0, x_i]$ is adjacent to $[0, x_j] \forall j, i \neq j, 1 \leq j \leq n-1$. Thus $L(C_I(N))$ is a *Complete Graph.* □

Remark 3.10. (i) Now provide an example to show that if N is not an integral nearring then for $I = \{0\}$ the graph $L(C_I(N))$ is not a complete graph. In Example 3.2(2), the nearring Z_6 not an integral nearring ($2 \cdot 3 = 0 \in Z_6$ and $2 \neq 0$ and $3 \neq 0$). Observe that $L(C_I(N))$ is not a *complete Graph.*
(ii) In Example 3.6, note that $I = \{0\}$ is not a c-prime ideal of $N(a \cdot c \in I$ but $a \notin I$ and $b \notin I)$. Observe that $L(C_I(N))$ is not a complete graph.

Proposition 3.11. *Let I be a c-prime ideal of simple integral nearring with $|N| = n$. Then $L(C_I(N))$ is a complete graph.*

Remark 3.12. Now provide an example that shows that if N is not an integral and simple nearring then for $I = \{0\}$ the graph $L(C_I(N))$ is not a complete graph. In Example 3.2(2), the nearring Z_6 is not a simple nearring ($I = \{0, 3\}$ is a c-prime ideal of Z_6.) Observe that $L(C_I(N))$ is not a *complete Graph.*

Proposition 3.13. *Let N be a simple integral nearring and I be a proper c-prime ideal of N. Then $gr(L(C_I(N))) = \infty$ if and only if $|N| = 2$ or $|N| = 3$.*

Proof. Let N be a *simple integral nearring.* Then only ideals of N are $\{0\}$ and N. Let I be the proper ideal of N.
$\Rightarrow$ Suppose that $grL((C_I(N))) = \infty$. Then $L(C_I(N))$ does not contain a cycle. Suppose $|N| > 3$. Then for $I = \{0\}$ get 0 is adjacent to every other vertex in $C_I(N)$. Then $L(C_I(N))$ contains a cycle, a contradiction. Also when $|N| > 3$ by Proposition 3.9, get $L(C_I(N))$ is a *complete graph* a contradiction. Hence $|N| = 2$ or $|N| = 3$.
$\Leftarrow$ Assume that $|N| = 2$ or $|N| = 3$. If $|N| = 2$ then $N = \{0, a\}$ for some $a \neq 0$. Then $(0, a)$ is an edge in $C_{\{0\}}(N)$ implies $[0, a]$ is the only one vertex in $L(C_{\{0\}}(N))$ implies $gr(L(C_I(N))) = \infty$. If $|N| = 3$ then $N = \{0, a, b\}$. Then $(0, a)$ and $(0, b)$ are the edges in $C_{\{0\}}(N)$.(Note that (a, b) is not an edge in $C_I(N)$ since N is integral and I is c-prime). Hence $[0, a]$ and $[0, b]$ are the only vertices in $L(C_{\{0\}}(N))$ implies $gr(L(C_I(N))) = \infty$. □

Remark 3.14. (1) Now provide an example show that if N is not an integral and simple nearring then $gr(L(C_I(N))) \neq \infty$ for $I = \{0\}$. In Example 3.2(2), the nearring Z_6 is not a simple nearring ($I = \{0, 3\}$ is a proper c-prime ideal of Z_6.) Observe that $gr(L(C_I(N))) \neq \infty$.

(2) Now provide an example to show that even when N is integral and simple, if $|N| > 3$ then $gr(L(C_I(N))) \neq \infty$.
Consider $N = Z_5$ be the ring of integers modulo 5. Note that N is integral and simple. For $I = \{0\}$ the graphs $C_I(N)$ and $L(C_I(N))$ is shown in Figure 7. Observe that $gr(L(C_I(N))) \neq \infty$.

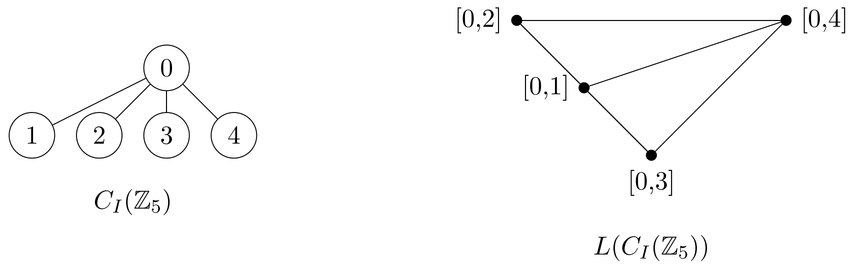


FIGURE 7. Diagram of $C_I(\mathbb{Z}_5)$ and $L(C_I(\mathbb{Z}_5))$

Proposition 3.15. *Let I be a 3-prime ideal of N. Let $a, b \in N$ such that $[a, b]$ be a vertex of $L(G_I(N))$. Then $a \in I$ or $b \in I$.*

Proof. If $a = 0$ or $b = 0$ the result is true (since 0 is in every ideal I of N). Let $a \neq 0$ and $b \neq 0$. Let $[a, b]$ be a vertex in $L(G_I(N))$. Then $(a, b) \in E(G_I(N))$ then by the definition of $G_I(N)$ get $aNb \subseteq I$ or $bNa \subseteq I$. Without loss of generality assume that $aNb \subseteq I$. As I is a *3-prime ideal*, get $a \in I$ or $b \in I$. □

Remark 3.16. If I is not a *3-prime ideal* of N then the above Proposition may not be true. Consider $N = \mathbb{Z}_6$ with $I = \{0\}$, which is not a *3-prime ideal.* We observe that $[3, 4]$ is a vertex in $L(G_I(N))$, $3 \notin I$ and $4 \notin I$.

Proposition 3.17. *Let I be a 3-prime ideal of N and $V(L(G_I(N))) = V(L(C_I(N)))$. Then I is a c-prime ideal of N.*

Proof. Let I be a *3-prime ideal* of N such that $V(L(G_I(N))) = V(L(C_I(N)))$. Let $a, b \in N$ such that $ab \in I$. Then by the definition of $C_I(N)$ get that (a, b) is an edge in $C_I(N)$ implies $[a, b]$ is a vertex in $L(C_I(N))$ implies $[a, b]$ is a vertex in $L(G_I(N))$ implies (a, b) is an edge in $G_I(N)$ implies $aNb \subseteq I$ or $bNa \subseteq I$. Without loss of generality may assume that $aNb \subseteq I$. As I is a *3-prime ideal* of N, get that $a \in I$ or $b \in I$. Thus I is a *c-prime ideal* of N. □

4. NEARRING HOMOMORPHISM AND GRAPH HOMOMORPHISM

Proposition 4.1. *Let $\theta : N_1 \to N_2$ be an onto nearring homomorphism, I be an ideal of N_1. Then θ is a graph-homomorphism from $L(G_I(N_1))$ to $L(G_{\theta(I)}(N_2))$.*

Proof. Let $a, b, c \in N$ such that $([a, b], [b, c])$ be an edge in $L(G_I(N_1))$. Then $aN_1b \subseteq I$ or $bN_1a \subseteq I$ and $bN_1c \subseteq I$ or $cN_1b \subseteq I$. Without loss of generality may assume that $aN_1b \subseteq I$ and $bN_1c \subseteq I$ implies $\theta(aN_1b) \subseteq \theta(I)$ and $\theta(bN_1c) \subseteq \theta(I)$. As θ is a nearring homomorphism, get that $\theta(a)\theta(N_1)\theta(b) \subseteq \theta(I)$ and $\theta(b)\theta(N_1)\theta(c) \subseteq \theta(I)$ implies $\theta(a)N_2\theta(b) \subseteq \theta(I)$ and $\theta(a)N_2\theta(c) \subseteq \theta(I)$ (Since θ is onto). We get $(\theta(a), \theta(b)) \in E(G_{\theta(I)}(N_2))$ and $(\theta(b), \theta(c)) \in E(G_{\theta(I)}(N_2))$ implies

$([\theta(a), \theta(b)], [\theta(b), \theta(c)]) \in E(L(G_{\theta(I)}(N_2)))$. Therefore θ is *Graph homomorphism* from $L(G_I(N_1))$ to $L(G_{\theta(I)}(N_2))$.

□

Proposition 4.2. *Let $\theta : N_1 \to N_2$ be a nearring homomorphism, I be an ideal of N_1. Then θ is a graph homomorphism from $L(C_I(N_1))$ to $L(C_{\theta(I)}(N_2))$.*

Proposition 4.3. *Let $\pi : N \to N/I$ be defined by $\pi(n) = n + I$ be a nearring epimorphism. Then π is a graph epimorphism from $L(C_I(N))$ to $L(C_{\pi(I)}(N/I))$.*

Proof. Let $a, b, c \in N$ such that $([a, b], [b, c]) \in E(L(C_I(N)))$. Then $ab \in I$ or $ba \in I$ and $cb \in I$ or $bc \in I$. Without loss of generality may assume that $ab \in I$ and $cb \in I$ then $\pi(ab) \in \pi(I)$ and $\pi(cb) \in \pi(I)$ implies $ab + I \in \pi(I)$ and $cb + I \in \pi(I)$ implies $(a + I)(b + I)) \in \pi(I)$ and $(c + I)(b + I) \in \pi(I)$ implies $\pi(a)\pi(b) \in \pi(I)$ and $\pi(c)\pi(b) \in \pi(I)$ implies $(\pi(a), \pi(b)) \in E(C_{\pi(I)}(N/I))$ and $(\pi(c), \pi(b)) \in E(C_{\pi(I)}(N/I))$ implies $([\pi(a), \pi(b)], [\pi(b), \pi(c)]) \in E(L(C_{\pi(I)}(N \setminus I)))$ Therefore π is a *Graph homomorphism* from $L(C_I(N))$ to $L(C_{\pi(I)}(N/I))$. □

Proposition 4.4. *Let I be a ideal and $a \in I, b \in I$. Then the set $P = \{[a, b]\}$ is a vertex dominating set of $L(C_I(N))$.*

Proof. From Theorem 3.10(i) form Kedukodi et al. [13], get every edge in $C_I(N)$ has one end in I. Hence I is an edge dominating set of $C_I(N)$. Then the edge set $\{(a, b) : a \in I, b \in I\}$ is an edge dominating set of $C_I(N)$. Hence $\{[a, b] : a \in I, b \in I\}$ is a vertex dominating set of $L(C_I(N))$. □

Remark 4.5. Now provide an example show that if $b \notin I$ then then $\{[a, b] : x \in I, b \notin I\}$ is not a vertex dominating set of $L(C_I(N))$.
In Example 3.2(2) in the nearring Z_6 and ideal $I = \{0, 3\}$ note that $2 \notin I$. Observe that $[0, 2]$ is not a vertex dominating set of $L(C_I(N))$.

Proposition 4.6. *Let $\theta : N_1 \to N_2$ be an onto nearring homomorphism, I be an ideal of N_1 and $a \in I, b \in I$. Then the set $P = \{[\theta(a), \theta(b)]\}$ is a vertex dominating set of $L(C_{\theta(I)}(N_2))$.*

Proposition 4.7. *Let $\theta : N_1 \to N_2$ be an onto nearring homomorphism, I be an ideal of N_1 and $a \in I, b \in I$. Then the set $P_2 = \{[\theta(a), \theta(b)]\}$ is a vertex dominating set of $L(G_{\theta(I)}(N_2))$.*

Proof. From Theorem 3.10(i) form Kedukodi et al. [13], get every edge in $G_I(N_1)$ has one end in I. Then the edge set $\{(a, b) : a \in I, b \in I\}$ is an edge dominating set of $G_I(N_1)$. By Proposition 4.2 get θ is a *graph homomorphism* from $L(G_I(N_1))$ to $L(G_{\theta(I)}(N_2))$. Hence $\{[\theta(a), \theta(b)] : a \in I, b \in I\}$ is a vertex dominating set of $L(G_{\theta(I)}(N_2))$. □

Definition 4.8. The *line graph* of $C_I(N \setminus I)$ is a graph with each edge of $C_I(N \setminus I)$ as vertex and any two distinct vertices are adjacent if and only if their corresponding edges share a common vertex in the graph of $C_I(N \setminus I)$.

Example 4.9. Consider $N = \mathbb{Z}_2 = \{0, 1\}$ and $I = \{0\}$ then $N \setminus I = \{1\}$, then $L(C_I(N))$ is an *empty graph*.
Consider $N = \mathbb{Z}_6$ and $I = \{0\}$ then $N \setminus I = \{1, 2, 3, 4, 5\}$ then $C_I(N \setminus I)$ and $L(C_I(N \setminus I))$ is shown in Figure 8.

$C_I(N \setminus I)$ $L(C_I(N \setminus I)$

FIGURE 8. Diagram of $C_I(N \setminus I)$ and $L(C_I(N \setminus I)$

Proposition 4.10. *Let I be an ideal of N.*
(i) If I is a 3-prime ideal of N then $L(G_I(N \setminus I))$ is an empty graph.
(ii) If I is a c-prime ideal of N then $L(C_I(N \setminus I))$ is an empty graph.

Proof. Let I be a 3-prime ideal of N. To prove (i), suppose $a, b \in N \setminus I$ such that (a, b) is an edge in $G_I(N \setminus I)$. Then $aNb \subseteq I$ or $bNa \subseteq I$. Without loss of generality assume $aNb \subseteq I$. As I is a 3-prime ideal of N get $a \in I$ or $b \in I$ a contradiction. Hence $G_I(N \setminus I)$ does not contain any edges. By definition of $L(G_I(N \setminus I))$ get $L(G_I(N \setminus I))$ is an empty graph.
The proof of (ii) is similar to proof of (i). □

Remark 4.11. Now provide an example to show that $L(G_I(N \setminus I))$ is not an empty graph when I is not a 3-prime ideal of N and $L(C_I(N \setminus I))$ is not an empty graph when I is not a c-prime ideal of N. In Example 4.9 note that $I = \{0\}$ not a 3-prime and not a c-prime ideal of N ($2N3 = \{0\}$ but $2 \notin I$ and $3 \notin I$ also $2 \cdot 3 = 0$ but $2 \notin I$ and $3 \notin I$.)Observe that $L(G_I(N \setminus I)) = L(C_I(N \setminus I))$ are not empty graph.

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